

# LOOK-UP-TABLE BASED DCT DOMAIN INVERSE MOTION COMPENSATION

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## ABSTRACT

DCT-based digital video coding standards such as MPEG and H.26x have been widely adopted for multimedia applications. Thus video processing in the DCT domain usually proves to be more efficient than in the spatial domain. To directly convert an inter-coded frame into an intra-coded frame in the DCT domain, the problem of *DCT domain inverse motion compensation* was studied in [1]. Since the data is organized block by block in the DCT domain, DCT domain inverse motion compensation is computationally intensive. In this paper, a look-up-table (LUT) based method for DCT domain inverse motion compensation is proposed by modeling the statistical distribution of DCT coefficients in typical images and video sequences. Compared to the method in [1], the LUT based method can save more than 50% of computing time based on experimental results. The memory requirement of the LUT is about 800 KB which is reasonable. Moreover, the LUT can be shared by multiple DCT domain video processing applications running on the same computer.

## 1. INTRODUCTION

With the emergence of video compression standards such as MPEG and H.26x, compressed digital video bit streams are widely used for high efficiency of storage and transmission. However, the compression schemes based on a combination of Discrete Cosine Transform (DCT) and Motion Compensation (MC) do not lead to easy manipulation and composition of the compressed video. There are two general approaches for processing compressed video bit streams: spatial domain processing and DCT domain processing. In spatial domain methods, the video bit stream is first fully decompressed to the spatial domain, then processed in the spatial domain, and finally re-compressed for storage or transmission. In DCT domain methods, the video bit stream is first partially decoded to the DCT domain, then processed in the DCT domain, and finally re-encoded. DCT domain

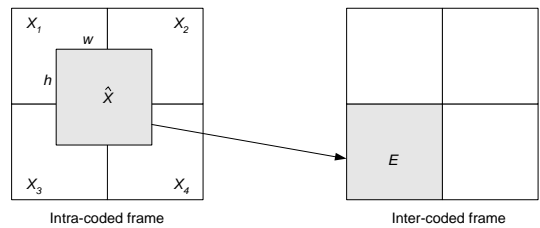


Fig. 1. DCT domain inverse motion compensation.

processing methods prove to be more efficient than spatial domain methods [1, 2] due to the following reasons: A) smaller data volume to be processed since DCT blocks are sparse; B) lower computational complexity due to elimination of the process of IDCT-DCT. In DCT domain video processing methods, DCT domain inverse motion compensation is usually used to convert an inter-coded frame into an intra-coded frame for video manipulation and composition. Since the data is organized block by block in the DCT domain, DCT domain inverse motion compensation has high computational complexity.

This problem was studied by Chang *et al.* [1]. The general setup is shown in Fig. 1, where  $\hat{x}$  can be expressed as a superposition of the appropriate windowed and shifted versions of  $x_1, x_2, x_3$  and  $x_4$ , *i.e.*,

$$\hat{x} = \sum_{i=1}^4 q_{i1} x_i q_{i2} \quad (1)$$

where  $q_{ij}, i = 1, \dots, 4, j = 1, 2$  are sparse  $8 \times 8$  matrices of zeros and ones that perform windowing and shifting operations. For example, for  $i = 1$ ,

$$q_{11} = \begin{pmatrix} O & I_h \\ O & O \end{pmatrix} \quad q_{12} = \begin{pmatrix} O & O \\ I_w & O \end{pmatrix} \quad (2)$$

where  $I_h$  and  $I_w$  are identity matrices of dimension  $h \times h$  and  $w \times w$ , respectively. The values  $h$  and  $w$  are determined by the motion vector of  $\hat{x}$ . By using the linear, distributive

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and unitary properties of DCT, it follows that

$$\hat{X} = \sum_{i=1}^4 Q_{i1} X_i Q_{i2} \quad (3)$$

where  $\hat{X}$ ,  $X_i$ ,  $Q_{i1}$  and  $Q_{i2}$  are the DCT's of  $\hat{x}$ ,  $x_i$ ,  $q_{i1}$  and  $q_{i2}$ , respectively.

Merhav *et al.* [2] proposed a fast algorithm to compute (3) by factorizing the fixed matrices  $Q_{ij}$  into a series of relatively sparse matrices instead of fully pre-computing them. Hence, some of the matrix multiplications can be avoided by simple addition and permutation operations. Assuncao *et al.* [3] approximated the elements of  $Q_{ij}$  by binary numbers with a maximum distortion of  $1/32$  so that all multiplications can be implemented by *shifts* and *additions*. They showed that in terms of operations (*shift*, *add*) required, their algorithm has only 28% of the computational complexity of the method proposed by Merhav *et al.* [2]. While all these methods utilize 2-D implementations, Acharya *et al.* [4] decomposed the problem into two separate 1-D problems. This decomposition proves to be more efficient than computing the combined operation. In (3), we have  $Q_{12} = Q_{32} \stackrel{\text{def}}{=} Q_{x0}$ ,  $Q_{22} = Q_{42} \stackrel{\text{def}}{=} Q_{x1}$  and  $Q_{11} = Q_{21} \stackrel{\text{def}}{=} Q_{y0}$ ,  $Q_{31} = Q_{41} \stackrel{\text{def}}{=} Q_{y1}$ . By using the separable scheme, (3) can be implemented as follows:

$$G_0 = X_1 Q_{x0} + X_2 Q_{x1} \quad (4)$$

$$G_1 = X_3 Q_{x0} + X_4 Q_{x1} \quad (5)$$

$$\hat{X} = Q_{y0} G_0 + Q_{y1} G_1 \quad (6)$$

where  $G_0$  and  $G_1$  are two intermediate blocks generated by horizontally shifting and windowing  $X_i, i = 1, 2, 3, 4$ , respectively.

In this paper, a look-up-table (LUT) based method for DCT domain inverse motion compensation is proposed by modeling the statistical distribution of DCT coefficients in typical images and video sequences. By pre-computing the multiplication results in (4 – 6) for those DCT coefficients with absolute value below a certain threshold, the LUT based method can save more than 50% of computing time, relative to Chang's algorithm [1]. The memory requirement of the LUT is about 800 KB which is reasonable based on the current computer memory capacity. Moreover, the LUT can be shared by multiple DCT domain video processing applications running on the same computer.

## 2. THE LUT BASED METHOD

### 2.1. Modeling distribution of DCT coefficients

All DCT coefficients in MPEG or H.26x coded images are quantized to integers with value ranging from -2048 to 2047. Since the DCT concentrates most of the signal energy into

relatively few coefficients, most AC coefficients have small values. The distribution of AC coefficients can be modeled as a Laplacian distribution with zero mean as follows [5, 6]:

$$p(x) = \frac{\lambda}{2} \exp(-\lambda|x|) \quad (7)$$

where

$$\lambda = \frac{1}{E[|X|]}. \quad (8)$$

Let  $\sigma^2$  be the variance of  $X$ , then  $\sigma = \frac{\sqrt{2}}{\lambda}$ . Given a positive threshold  $TH$ , one can have

$$\begin{aligned} P(|X| \leq TH) &= \int_{-TH}^{TH} p(x) dx \\ &= \int_0^{TH} \lambda \exp(-\lambda x) dx \end{aligned} \quad (9)$$

A few JPEG-coded images and I frames from several MPEG-coded video sequences are selected to estimate the value of  $\lambda$  according to (8). As a result, we obtain  $\lambda \approx 0.0284$ . If we set a threshold  $TH = 2\sigma \approx 100$ , then more than 94% of AC coefficients have absolute value smaller than the threshold  $TH$  according to (9). This implies that a lot of computation can be saved by pre-computing the multiplication results for all those coefficients having absolute value smaller than the threshold  $TH$ . In the following, the implementation of the LUT based method will be discussed.

### 2.2. LUT design

By using the separable approach [4], only 1-D case needs to be considered to build the LUT. So only two tables are needed to save all multiplication results with  $Q_{x0}$  and  $Q_{x1}$  in (4) and (5). In Fig. 1,  $w$  has 16 possible values including half-pixel resolution, *i.e.*  $w = 0, 0.5, 1, \dots, 7.5$ . For 1-D case, each non-zero element in  $\{X_i\}$  contributes to eight entries of  $G_0$  or  $G_1$ . As a result, a four dimensional table is needed to save the pre-computed results, *i.e.*  $\text{Table}[v][p][w][i]$ , where  $v$  represents the absolute value of DCT coefficients,  $p$  represents the column position of DCT coefficients in  $\{X_i\}$ ,  $w$  is shown in Fig. 1 and  $i$  represents the column position of the pre-computed results in the target block. Both  $p$  and  $i$  have eight possible values each.  $v$  has 100 possible values since  $TH$  is 100. If four bytes are used to store each entry of the table, the size of table is

$$\text{size} = 4 \times v \times p \times w \times i = 4 \times 100 \times 8 \times 16 \times 8 = 400KB. \quad (10)$$

Hence, the total memory requirement for two tables is about 800KB when  $TH = 100$ . This is reasonable based on the current computer memory capacity. The vertical operations in (6) can be converted to the horizontal operations via matrix transposition so that the LUT can be reused.

### 2.3. DC coefficients

In DCT-coded images, the distribution of DC coefficients has a large mean value (*e.g.* 1000) and a larger variance relative to the distribution of AC coefficients as shown in Fig. 2. Since the LUT is created by modeling the distribution of AC coefficients, it doesn't apply to DC coefficients because most DC coefficients are much larger than the threshold  $TH$ . For adjacent DCT blocks, the DC coefficients are highly correlated in typical images [7]. Therefore, the difference between adjacent DC coefficients should have much smaller mean value and dynamical range than the DC coefficient itself. With this observation, (4) can be rewritten as

$$\begin{aligned} G_0 &= X_1 Q_{x0} + X_2 Q_{x1} \\ &= \frac{X_1 + X_2}{2} (Q_{x0} + Q_{x1}) + \\ &\quad \frac{X_1 - X_2}{2} (Q_{x0} - Q_{x1}). \end{aligned} \quad (11)$$

Let  $Q_+ = Q_{x0} + Q_{x1}$ ,  $Q_+$  has the property:

$$Q_+(0, 0) = 1; Q_+(0, j) = Q_+(j, 0) = 0; j = 1, \dots, 7.$$

This means that the summation of the DC components of  $X_1$  and  $X_2$  only contributes to the DC component of  $G_0$ . So we can just sum up both DC coefficients from  $X_1$  and  $X_2$ , then fill it in the DC entry of  $G_0$  without any further computation. The difference between the DC component of  $X_1$  and that of  $X_2$  has distribution similar to the distribution of AC coefficients as shown in Fig. 3. In the selected JPEG-coded images, more than 70% of the difference values have absolute value below the threshold  $TH$  so that their multiplication results in (11) can be directly obtained from the LUT.

### 3. EXPERIMENTAL RESULTS

Both Chang's method [1] and the LUT based method were implemented. Four MPEG-coded video sequences with intensive motion activities were selected for our experiments, *i.e.*, "Foreman", "Coastguard", "Mobile" and "Stefan". All sequences are CIF resolution with 352 pixels and 288 lines. All P and B frames in the selected video sequences were converted into I frames by using both methods. In the LUT based method, for those DCT coefficients with absolute value greater than the threshold  $TH$ , Chang's method was employed to compute the corresponding multiplication results. The time for reconstructing one P or B frame to an I frame for both methods was measured on a Windows NT workstation with 300 MHz CPU and 512 MB memory, respectively. The average time for converting one P or B frame into an I frame is tabulated in Table 1. On average, the LUT based method saved the computing time by more than 70%

| Video sequence | Chang's method |         | The LUT based method |         |
|----------------|----------------|---------|----------------------|---------|
|                | P frame        | B frame | P frame              | B frame |
| "Foreman"      | 0.3137         | 0.4738  | 0.0931               | 0.1423  |
| "Coastguard"   | 0.2374         | 0.3417  | 0.0912               | 0.1190  |
| "Mobile"       | 0.3487         | 0.4136  | 0.1462               | 0.2000  |
| "Stefan"       | 0.2057         | 0.3667  | 0.0780               | 0.1416  |

**Table 1.** The average computing time to convert one P or B frame to an I frame (Unit: second)

in "Foreman", 60% in "Coastguard" and "Stefan" and 50% in "Mobile" according to the results in Table 1. In Fig. 4, the time for reconstructing each P frame to an I frame in the sequence "Mobile" is plotted for both methods. Fig. 5 shows the time for each B frame. As can be seen, the computing time in the LUT based method was almost constant for different P or B frames while the computing time in Chang's method changes dramatically. One reason is that for motion vectors with half-pixel precision, the complexity of Chang's method is doubled due to the interpolation, while it is constant in the LUT method because the results for half-pixel resolution are pre-computed and saved in the LUT.

### 4. CONCLUSION

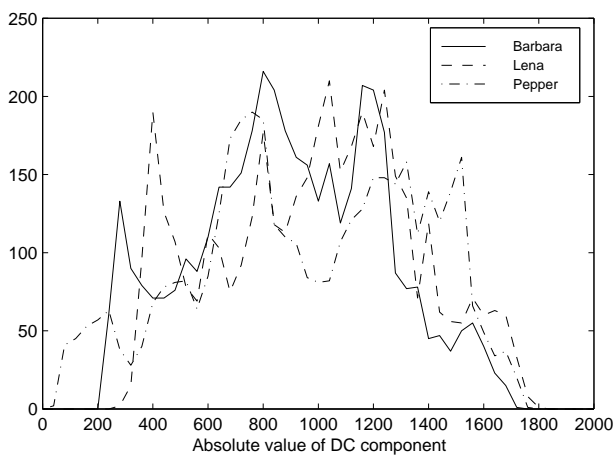
In this paper, a LUT based method for DCT domain inverse motion compensation is proposed by modeling the distribution of DCT coefficients in typical images and video sequences. Relative to Chang's method, more than 50% of computing time can be saved based on the experimental results. Compared to other fast algorithms [2, 3], the LUT based method is straightforward to implement and introduces no error. Moreover, for motion vectors with half-pixel precision, the algorithms in [2, 3] have the same problem as Chang's algorithm, *i.e.*, the computational complexity is doubled. However, the LUT method has no such problem since all corresponding results are pre-computed and saved in the LUT. This can help reduce the jerkiness in video manipulation and composition applications. The memory requirement of the LUT is about 800 KB which is reasonable based on the current computer memory capacity. In addition, the LUT can be shared by multiple video processing applications running on the same machine. For example, in a video editing server, usually multiple processes are running simultaneously to process different video bit streams. Only one LUT is needed for these parallel processes to conduct DCT domain inverse motion compensation.

### 5. REFERENCES

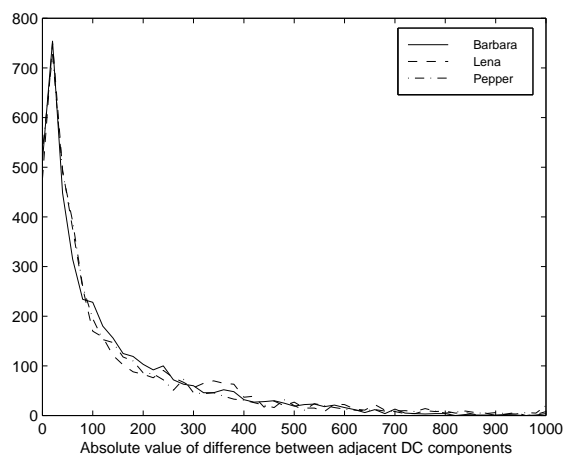
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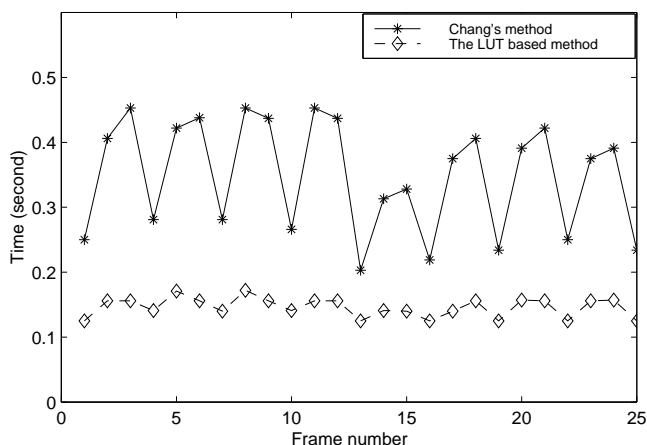
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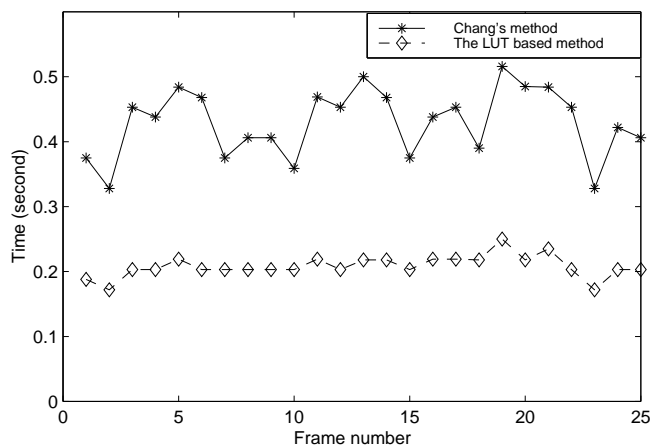
**Fig. 2.** Histograms of DC coefficients in images with the Bin size 40.



**Fig. 3.** Histograms of difference between adjacent DC coefficients in images with the Bin size 20.



**Fig. 4.** The computing time for reconstructing each P frame to I frame in the video sequence “Mobile”.



**Fig. 5.** The computing time for reconstructing each B frame to I frame in the video sequence “Mobile”.